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ZARISKI-FINITE MODULES

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ABSTRACT. Let R be a commutative ring with identity, and let M be an R-module. We examine the situation where the Zariski space of M is finitely generated. In case R is a Noetherian UFD and M is finitely generated and nontorsion, we prove that this condition is equivalent to M being isomorphic to a direct sum of R and a finite R-module. We further describe in this case the structure of all but finitely many prime submodules of M.

Zariski spaces of modules were introduced in [**8**], and they provide a demonstration that the study of prime submodules is significantly more rich and complex than is the study of prime ideals. A Zariski space is determined by the following. Let R be a commutative ring with identity, and let M be a unital R-module. Note that the Zariski topology on $\operatorname{spec} R$, denoted $\zeta(\mathcal{R})$, is a semiring, where "addition" is given by intersection and "multiplication" is given by union. In a like manner, we let $\zeta(M)$ be the collection of all varieties of submodules of M, and we observe that although $\zeta(M)$ is not in general a topology itself, it is a semimodule over $\zeta(R)$, where "addition" and "scalar multiplication" are given by

$$V(L) + V(N) = V(L) \cap V(N) = V(L + N)$$

and

$$V(\mathfrak{a})V(N) = V(\mathfrak{a}N)$$

for all submodules L and N of M and for all ideals $\mathfrak{a}$ of R.

This note concerns the generating of the semimodule $\zeta(M)$ as "linear combinations" of its elements, particularly finitely many of them. For a basic study of semimodules, see, for example, [**3**].

Now in the case that M is a finitely generated, noncyclic, faithful multiplication module over R, note that although the structure of M is

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