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STABILITY OF DIFFEOMORPHISMS ALONG ONE PARAMETER

MING-CHIA LI

ABSTRACT. The structural stability theorem, proved by Robbin [**6**] and Robinson [**7**], says that for an Axiom A diffeomorphism f with the strong transversality condition, there exists a sufficiently small neighborhood U of f in the set of C^1 diffeomorphisms such that if $g \in U$ then there is a homeomorphism h near the identity map such that f is conjugate to g, i.e., $hf = gh$.

In this paper we further investigate the size of the neighborhood U and the distance of the homeomorphism h with the identity map. We show that if $\{f_\varepsilon\}$ is a one-parameter family of C^3 diffeomorphisms, f_0 satisfies Axiom A and the strong transversality condition, and f_ε is C^0 $O(\varepsilon^3)$-close and C^1 $O(\varepsilon^2)$-close to f_0, then for all small $|\varepsilon|$, there is a homeomorphism h_ε with C^0 $O(\varepsilon^2)$ near the identity map, such that $h_\varepsilon f_0 = f_\varepsilon h_\varepsilon$.

1. Definitions and the main theorem. First of all, we introduce notations and basic definitions.

Throughout this paper, let M denote a smooth compact manifold with a distance d induced from the Riemannian metric, d_{C^0} denote a distance in the set of continuous maps on M with the standard C^0-topology, and d_{C^1} denote a distance in the set of C^1 diffeomorphisms on M with the strong C^1-topology. For $r = 0$ or 1, $p \in \mathbf{N}$, we say that f is C^r $O(\varepsilon^p)$ to g if the ratio $|d_{C^r}(f, g)/\varepsilon^p|$ is bounded as $\varepsilon \to 0$.

A compact invariant set Λ for a diffeomorphism f on M has a *hyperbolic structure* if $TM|_\Lambda$, the restriction of the tangent bundle TM of M to Λ has two subbundles $\mathbf{E}^s$ and $\mathbf{E}^u$ such that $TM|_\Lambda = (\mathbf{E}^s \oplus \mathbf{E}^u)|_\Lambda$ where $\oplus$ is the Whitney sum of two subbundles, and if there exist $C > 0$ and $0 < \mu < 1$ such that, for any $x \in M$ and for all

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