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COEXISTENCE OF CENTERS AND LIMIT CYCLES IN POLYNOMIAL SYSTEMS

ROBERT E. KOOIJ AND ANDRÉ ZEGELING

ABSTRACT. In this paper we consider polynomial systems on the plane with coexisting centers and limit cycles. We prove that cubic systems which are symmetric with respect to a line which is not invariant have either zero or exactly two limit cycles. The same result is proved for rationally reversible cubic systems. New configurations of coexisting centers and limit cycles in cubic systems are presented. Also an example of an integrable quartic system with a unique limit cycle and a center is given. As a bonus we construct a septic system with 57 limit cycles.

1. Introduction. It is a well-known result that a quadratic system with a center has no limit cycles, see Ye [**20**]. However, Borukhov [**3**] and Dolov [**6**] gave examples of quartic and cubic systems with both centers and limit cycles. In this paper we give new configurations of coexisting centers and limit cycles in polynomial systems. The existence of centers in the examples for cubic systems is proved by using a symmetry principle. In Section 3 we derive some results for cubic systems symmetric with respect to a line, also known as time-reversible systems. The main result is that these systems, if the line is not invariant, have either zero or two limit cycles (Theorem 3.1). Using the results of Section 3, we present in Section 4 an example of a cubic system with three centers and two limit cycles where each limit cycle surrounds exactly one singularity. In a similar fashion in Section 5 an example is given of a cubic system with one center and two limit cycles where each limit cycle surrounds three singularities. In Section 6 we show that all rationally reversible cubic systems have either zero or two limit cycles (Theorem 6.1). This generalizes the result of Section 3. In Section 7 we give an example of a quartic system with a unique limit cycle and center. Here the existence of the center is not proved using a symmetry principle but by calculating the first integral

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