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ON HAPPY NUMBERS

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ABSTRACT. The happy function $T : \mathbf{N} \to \mathbf{N}$ is the mapping which sends a natural number to the sum of the squares of its decimal digits. A happy number x is a natural number for which the sequence $\{T^n(x)\}_{n=1}^{\infty}$ eventually reaches 1. Guy asks in [**1**], problem E34, if there exist sequences of consecutive happy numbers of arbitrary length. In this paper we answer this question affirmatively.

1. Introduction. In [**1**, problem E34, p. 234], a happy number is defined as follows:

> If you iterate the process of summing the squares of the decimal digits of a number, then it is easy to see that you either reach the cycle
>
> $$4 \to 16 \to 37 \to 58 \to 89 \to 145 \to 42 \to 20 \to 4$$
>
> or arrive at 1. In the latter case you started from a happy number.

Several questions are asked about happy numbers, including: "How many consecutive happy numbers can you have? Can there be arbitrarily many?" It is the purpose of this paper to show that there are sequences of consecutive happy numbers of arbitrary length. It is hoped that our proof will be a model for answering similar questions involving iterates of maps depending on the digital representations of natural numbers.

2. Some preliminary results. We gather here some preliminary results which will be needed in our proof of the existence of sequences of consecutive happy numbers of arbitrary length. These are all elementary except for the final lemma, which affirms the existence of

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