ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 2, Summer 2000

EXACT SOLUTIONS OF A CLASS OF DIFFERENTIAL EQUATIONS OF LAMÉ'S TYPE AND ITS APPLICATIONS TO CONTACT GEOMETRY

BANG-YEN CHEN

ABSTRACT. The study of linear differential equations with one or more of their coefficients involving Jacobi's elliptic functions was initiated by Picard. Among such linear differential equations perhaps the most famous one is the equation of Lamé. The methods of finding the exact solutions of the Lamé equation have been investigated by many mathematicians. In this note we investigate a class of differential equations of Lamé's type which arise naturally in the study of Legendre curves in contact geometry. We present the exact solutions of this class of differential equations and apply them to determine explicitly the Legendre curves associated with the exact solutions of this class of differential equations.

1. Introduction. The study of linear differential equations with coefficients involving uniform doubly periodic functions of the independent variable was initiated by Picard. For instance, Picard had shown that every linear differential equation with uniform doubly periodic coefficients and possessing only uniform solutions has always at least one solution which is a doubly periodic function of the second kind. Among linear differential equations with uniform doubly periodic coefficients perhaps the most famous one is the equation of Lamé:

$$\frac{d^2y}{dx^2} = [n(n+1)k^2\mathrm{sn}^2(x,k) + c]y.$$

The methods of finding the exact solutions of the Lamé equation have been studied by many mathematicians. For a recent study on Lamé's equation and its applications to physics, see, for instance, [**4**].

Legendre curves are known to play an important role in the study of contact manifolds, e.g., a diffeomorphism of a contact manifold

Received by the editors on October 26, 1997, and in revised form on January 14, 1999.
1991 AMS *Mathematics Subject Classification.* Primary 53A04, Secondary 34A05.
Key words and phrases. Legendre curve, contact geometry, exact solutions, differential equations of Lamé type.