ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 2, Summer 2000

STABILITY OF BROCARD POINTS OF POLYGONS

ADI BEN-ISRAEL AND STEPHAN FOLDES

ABSTRACT. A continuous nested sequence of similar triangles converging to the Brocard point of a given triangle is investigated. All these triangles have the same Brocard point. For polygons, the Brocard point need not exist, but there is always a limit object for an analogously defined nested sequence of inner polygons. This limit object is a Brocard point if and only if the inner polygons are all similar to the original polygon. The similarity of two distinct inner polygons already suffices. In that case, all the inner polygons have the same Brocard point.

1. Introduction. The positive Brocard point of a triangle $A_1A_2A_3$ is the unique point Ω within the triangle such that the angle between $A_i\Omega$ and A_iA_{i+1} is the same for all i modulo 3. This is illustrated in Figure 2, where the vertices are denoted A, B, C. The earliest easily accessible reference to the Brocard point that we are aware of is [**16**]. According to Honsberger [**6**], the Brocard point was already known to Crelle, Jacobi and others at the beginning of the 19th century. Indeed, the historically more accurate name of *Crelle-Brocard point* is used by Mitrinovic, Pecaric and Volenec [**15**] (where other references to both older and contemporary work are also given).

Traditionally, the Brocard point was constructed by rule and compass: see Honsberger [**6**], Johnson [**7**], Shively [**17**]. An entirely different approach to generate the Brocard point, by an infinite limit process, was taken by Yff in [**18**]. Another infinite limit process to generate the Brocard point was described by the present authors in [**1**]. In this latter paper the limit process was defined for arbitrary convex polygons, and it yields the Brocard point whenever it exists, as in Figure 3. (For n-gons the Brocard point is defined analogously with the triangle case as

Received by the editors on January 4, 1999, and in revised form on May 18, 1999.
1991 AMS *Mathematics Subject Classification.* Primary 51M15, 51M20, Secondary 51M04, 52C05.
Key words and phrases. Triangle geometry, Brocard angle, Brocard points, convex polygons, similarity.
The work of the second author was partially supported by ONR grant N00014-92-J-1375 and by RUTCOR.