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TURÁN INEQUALITIES FOR SYMMETRIC ASKEY-WILSON POLYNOMIALS

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1. Let $\{P_n(x) : n = 0, 1, \dots\}$ be a sequence of polynomials orthogonal on an interval $[a, b]$. The polynomials $\{P_n(x)\}$ are said to satisfy Turán's inequality if

$$P_n^2(x) - P_{n+1}(x)P_{n-1}(x) \geq 0, \quad a \leq x \leq b, \quad n = 0, 1, \dots . \tag{1.1}$$

Turán first observed that (1.1) is satisfied by Legendre polynomials [**9**] and Szego [**8**] gave two beautiful proofs of that fact. Various authors have generalized (1.1) to the classical orthogonal polynomials of Jacobi, Hermite, and Laguerre [**1**], [**5**]. Szasz [**7**] also proved a Turán inequality for ultraspherical polynomials and Bessel functions.

Bustoz and Ismail [**4**] applied a procedure first used by Szász [**7**] to prove Turán inequalities for an important class of nonclassical orthogonal polynomials; the symmetric Pollaczek polynomials as well as for modified Lommel polynomials, and for q-Bessel functions. Also in [**3**] Bustoz and Ismail proved a Turán inequality for continuous q-ultraspherical polynomials by using the Szász technique. In this paper we will apply the Szász technique to prove a Turán inequality for symmetric Askey-Wilson polynomials.

2. Askey-Wilson polynomials. The q-shifted factorial $(a;q)_n$ is defined by

$$(a;q)_n = \begin{cases} 1 & n = 0 \\ (1-a)(1-aq)\cdots(1-aq^{n-1}) & n = 1, 2, \dots, \end{cases}$$

and for $|q| < 1$ we define

$$(a;q)_\infty = \prod_{j=0}^{\infty}(1 - aq^j).$$

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