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ANDERSON'S CONJECTURE FOR DOMAINS WITH FRACTAL BOUNDARY

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ABSTRACT. The inequality

$$\liminf_{r\to 1} \frac{\operatorname{Re} b(r\zeta)}{\int_0^r |b'(p\zeta)|\, d\rho} > 0$$

is shown to hold for all ζ in a set $E \subset \mathbf{T}$ with Hausdorff dimension 1, when b lies in a special class of Bloch functions first considered by Jones.

1. Introduction and background. A function f, defined and analytic in the unit disk, is called a Bloch function if

$$\|f\|_{\mathcal{B}} = \sup_{z\in\mathbf{D}} (1-|z|^2)|f'(z)| < \infty.$$

We write $f \in \mathcal{B}$. The following proposition, which establishes a close connection between Bloch functions and conformal mappings, is well known, see [**2, 3**].

Proposition 1.1. *If g is a univalent function in $\mathbf{D}$ and $f = \log g'$, then $f \in \mathcal{B}$ and $\|f\|_{\mathcal{B}} \le 6$. Conversely, if $\|f\|_{\mathcal{B}} \le 1$, then there exists a univalent function g such that $f = \log g'$.*

Functions in the Bloch space are Lipschitz mappings from the disk with the hyperbolic metric to the complex plane with the Euclidean metric

$$|b(z_1) - b(z_2)| \le C\|b\|_{\mathcal{B}} d(z_1, z_2).$$

This is easily seen by integration because the hyperbolic distance between two points z_1 and z_2 in the unit disk is defined as

$$d(z_1, z_2) = \inf_{\gamma} \int_{\gamma} \frac{2|dz|}{1-|z|^2}$$

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