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ASYMPTOTIC ANALYSIS OF DISSOLUTION OF A SPHERICAL BUBBLE (CASE OF FAST REACTION OUTSIDE THE BUBBLE)

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ABSTRACT. This paper analyzes the dissolution of a spherical gas bubble as the gas diffuses out of the bubble into a liquid and is consumed by a fast reaction in the liquid. Due to the fast reaction a small parameter ε appears in the problem formulation which makes the problem singularly perturbed. The boundary function method is used to derive uniform asymptotic approximations to the bubble's radius and gas concentration in the liquid. Existence, and uniqueness of the solution, as well as the asymptotic correctness of the approximations are shown.

1. Introduction. We consider a gas bubble moving through a liquid in a bubble reactor and shrinking as the gas is consumed in a reaction occurring near the surface of the bubble. In modeling different types of bubble reactors, catalytic bubble reactors, etc., the, so-called, film model [**2**] is widely used. According to this model it is assumed that the reactions accompanied by rapid spatial changes in concentration of the reacting components occur only in a 'thin film' near the surface of the bubble whereas concentrations of the reacting substances in the bulk of the liquid and inside the bubbles are constant in space and only change in time. In more complicated models, involving also the catalyst particles, the presence of two types of films is assumed: one near the surface of gas bubbles, and the other near the surface of catalyst particles. As of now, in practical calculations chemical engineers do not take into account the process of shrinking of the bubbles as they move through the reactor (the gas/liquid ratio is one of the parameters playing an important role in the reactor design, and it is assumed to be constant throughout the reactor in practical calculations [**4**]).

In this paper we attempt a more realistic approach that, in the case of fast reactions, allows us to eliminate the 'film' assumption. The concentrations will now be nonconstant in the bulk liquid but we

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