ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 1, Spring 2000

A STRONG SIMILARITY PROPERTY OF NUCLEAR C^*-ALGEBRAS

CHRISTIAN LE MERDY

1. Introduction and main result. The aim of this note is to establish a new lifting property of the multiplication map on nuclear C^*-algebras, Theorem 1.2 below, and to apply it to two natural questions arising from Pisier's recent work on the similarity problem for operator algebras [**19**]. Let A be a C^*-algebra, H a Hilbert space, and let $u : A \to B(H)$ be a bounded homomorphism. An outstanding open problem going back to Kadison asks whether u is necessarily similar to a $*$-representation. By a result due to Paulsen [**14**], this is equivalent to the question: Is u automatically completely bounded? We refer to [**15, 17**] for wide information on completely bounded maps and Kadison's similarity problem.

When A is a nuclear C^*-algebra, Kadison's problem was solved positively by Bunce [**3**] and Christensen [**6**]. Moreover, in this situation we have the estimate $\|u\|_{\mathrm{cb}} \leq \|u\|^2$ for any bounded homomorphism u from A into $B(H)$ where $\|\cdot\|_{\mathrm{cb}}$ denotes the completely bounded norm. In [**19**], Pisier showed that this estimate is not far from characterizing nuclear C^*-algebras. Firstly he proved that if A is a C^*-algebra for which any bounded homomorphism $u : A \to B(H)$ is completely bounded, there exists a number $\alpha \geq 0$ and a constant $K > 0$ such that, for all u as above, $\|u\|_{\mathrm{cb}} \leq K\|u\|^{\alpha}$. Moreover, he showed that the infimum of the numbers $\alpha \geq 0$ for which this holds is attained and is an integer. This integer is denoted by $d(A)$ and called the similarity degree of A. With this terminology, we thus have $d(A) \leq 2$ when A is a nuclear C^*-algebra. Secondly it is shown in [**19**] that if A is a C^* algebra with $d(A) \leq 2$, then whenever a $*$-representation $\pi : A \to B(H)$ generates a semi-finite von Neumann algebra, that von Neumann algebra is injective.

Our first purpose is to show that the degree 2 property of nuclear C^*-algebras actually holds in a strong sense, as follows. Let A be a

Received by the editors on September 1, 1997, and in revised form on October 30, 1998.