ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 1, Spring 2000

MIDDLE SEMICONTINUITY FOR UNBOUNDED OPERATORS

HYOUNGSOON KIM

ABSTRACT. Let A be a C^*-algebra and K_A its Pedersen's ideal. By making use of Mack's characterization of PCS-algebra and Phillips' new description of multipliers of K_A, [**14, 18**], we generalize the concept of middle semicontinuity [**6**] to the case of unbounded operators affiliated with A^{**}, the enveloping von Neumann algebra of A. Especially we obtain the unbounded version of a Dauns-Hofmann type theorem [**15**, Theorem 4.6] and a middle interpolation theorem [**6**, Theorem 3.40].

1. Introduction and preliminaries. Let A be a C^*-algebra and A^{**} its enveloping von Neumann algebra. The theory of semicontinuous operators in A^{**} was developed by Pedersen, Akemann and Brown [**2, 6, 15**]. This paper is a sequel to [**12**] which generalizes the theory of strong semicontinuity. We will adopt the same notations from it. In this paper the concept of middle semicontinuity is generalized for unbounded operators affiliated with A^{**}.

Let $M(A)$ denote the multiplier algebra of A and K_A the Pedersen's ideal (minimal dense ideal) of A. If A is commutative, that is, $A = C_0(X)$, the algebra of all complex valued continuous functions which vanish at infinity on some locally compact space X, then $M(A)$, respectively K_A, can be identified with $C_b(X)$, respectively $C_c(X)$, the algebra of all complex value bounded, respectively compactly supported, continuous functions on X. As a noncommutative generalization of the relation between $C_c(X)$ and its multiplier algebra $C(X)$, Lazar and Taylor [**13**] introduced $\Gamma(K_A)$, the multipliers (double centralizers) of Pedersen's ideal K_A and made an extensive study of it.

In [**18**], Phillips gave a new description of $\Gamma(K_A)$ as an inverse limit of C^*-algebras (pro C^*-algebra) and derived a number of the results of [**13**] directly from corresponding facts about inverse limits of C^*-algebras.

Received by the editors on May 3, 1997, and in revised form on September 22, 1998.