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A GENERALIZATION OF A CONJECTURE OF HARDY AND LITTLEWOOD TO ALGEBRAIC NUMBER FIELDS

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ABSTRACT. We generalize conjectures of Hardy and Littlewood concerning the density of twin primes and k-tuples of primes to arbitrary algebraic number fields.

In one of their great *Partitio Numerorum* papers [**7**], Hardy and Littlewood advance a number of conjectures involving the density of pairs and k-tuples of primes separated by fixed gaps. For example, if d is even, we define

$$P_d(x) = |\{0 < n < x : n, n+d \text{ are both prime}\}|.$$

They conjecture both that

$$\lim_{x\to\infty} \frac{P_d(x)}{P_2(x)} = \prod_{\text{odd } p|d} \frac{p-1}{p-2}$$

and that $P_2(x)$ is asymptotic to

$$2\prod_{p>2}\left(1-\frac{1}{(p-1)^2}\right)\int_2^x \frac{dy}{(\log y)^2}.$$

We will refer to the first equation as the "relative conjecture" and the second as the "absolute conjecture."

There has been much numerical verification of these conjectures and many attempts at proofs. Balog [**1**] proves a result that implies that the conjectures are true "on average," where the average is taken over the possible shapes of the k-tuples. Golubev [**6**] compares these conjectures

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