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SMOOTH POINTS OF ESSENTIALLY BOUNDED VECTOR FUNCTION SPACES

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ABSTRACT. We characterize the smooth points of $L_\infty(X)$, where X is any normed space.

1. Introduction. Let X be a normed space and $x, y \in X$. The one-sided derivatives at $x \neq 0$ in the direction $y \neq 0$ are

$$D_X^{\pm}(x,y) = \lim_{h\to 0^{\pm}} \frac{\|x+hy\| - \|x\|}{h}.$$

Both limits always exist and, if they have the same value, we write $D_X(x,y) = D_X^+(x,y) = D_X^-(x,y)$. It is easy to see that this is equivalent to saying: For every $\varepsilon > 0$ there exists $\delta > 0$ such that $0 < h < \delta$ implies $\|x+hy\| + \|x-hy\| < 2\|x\| + \varepsilon h$.

We say that $x \neq 0$ is *smooth*, if $D(x,y)$ exists, for every $y \in S_X$, where S_X denotes the unit sphere of X, or equivalently, if there is a unique norm-one functional $x^* \in X^*$, the topological dual of X, such that $x^*(x) = \|x\|$ [**1**, page 179]. Since $D_X(tx,y) = D_X(x,y)$ for $t > 0$, we can restrict our attention to the smooth points of S_X.

Deeb and Khalil [**3**] have characterized the smooth points of the Lebesgue-Bochner spaces $L_p(I,X)$, $1 \leq p < \infty$, when I has finite measure and X has a separable dual. Cerda, Hudzik and Mastylo [**2**] characterize the smooth points of the Köthe-Bochner space $E(X)$, if X is real with separable dual, E is order continuous, and the norm of E^* is strictly monotonic. In this paper we characterize the smooth points of $L_\infty(X)$. In contrast to the $L_p(I,X)$, $1 \leq p < \infty$, it is worth noticing that the smoothness of $x \in L_\infty(X)$ does not imply the smoothness of $x(t) \in X$ for almost every $t \in T$.

Let (T, Σ, μ) be a complete, positive measure space and X a normed space. The function $x : T \to X$ is said to be *simple* if there

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