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LOCALNESS OF THE CENTRALIZER NEARRING DETERMINED BY End G

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ABSTRACT. For G a finite p-group, we investigate the localness of the nearring $M_E(G) = \{f : G \to G \mid f\sigma = \sigma f$ for every $\sigma \in \operatorname{End} G\}$. Examples of groups which make $M_E(G)$ local are provided.

1. Introduction. Let G be a group written additively but not necessarily abelian, and let S be a subsemigroup of $\operatorname{End} G$. The set $M_S(G) = \{f : G \to G \mid f\sigma = \sigma f$ for every $\sigma \in S\}$ forms a nearring under pointwise addition and function composition and is called the *centralizer nearring determined by S and G*. These nearrings are very general since every nearring with identity is isomorphic to an $M_S(G)$ for some pair S and G [**5**, 14.3]. Therefore, it is difficult to investigate these nearrings without some restriction on either G or S. In particular, much attention has been focused on the case where S is a group of automorphisms of G (e.g., see [**6**] or [**9**]).

If S consists of only the identity function on G, then $M_S(G) = M(G)$, the set of all functions from G to G. Similarly, if S consists of only the zero function on G, then $M_S(G) = M_0(G)$, the set of all zero-preserving functions from G to G. The structure of these nearrings is well known (see [**5, 11**] or [**12**] for information and for other general results about nearrings). In this paper which contains results from the author's doctoral dissertation [**2**], we are interested in the structure of the other extreme situation, in other words, when $S = \operatorname{End} G = E$. We call $M_E(G)$ the *centralizer nearring determined by* $\operatorname{End} G$. Since $\operatorname{End} G$ contains the zero function, $M_E(G)$ will be a zero-symmetric nearring.

We recall that a nearring N is local if the set of nonunits in N forms an additive subgroup. If N is finite, this condition is equivalent to saying that every element of N is either invertible or nilpotent [**10**]. This provides further motivation for studying $M_E(G)$, for if G is finite and $M_E(G)$ is not local, then $M_S(G)$ cannot be local for any subsemigroup

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