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DUALITY OF THE BERGMAN SPACES ON SOME WEAKLY PSEUDOCONVEX DOMAINS

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1. Introduction and statement of results. Let D be a smoothly bounded pseudoconvex domain of finite type in $\mathbf{C}^2$, i.e., every one-dimensional complex submanifold of $\mathbf{C}^2$ which is tangent to the boundary bD has finite order of contact with bD. There are many equivalent formulations of finite type. For a more complete description the reader is referred to the survey article by D'Angelo [**6**] and the references therein.

For $1 \leq p < \infty$ and dV Lebesgue measure we consider the closed subspace $\mathcal{O}L^p(D)$ of $L^p(D, dV)$ consisting of holomorphic functions. These spaces are commonly referred to as the Bergman spaces. $\mathcal{O}L^p(D)$ is a Banach space with norm $\|f\|_p^p = \int_D |f(z)|^p \, dV(z)$. As usual, if X is a Banach space, we denote its dual by X^*. X^* is also a Banach space with norm $\|\cdot\|^*$. In this paper we prove the following

Theorem A. *For $1 < p < \infty$, the dual of $\mathcal{O}L^p(D)$ can be identified with $\mathcal{O}L^q(D)$ where $(1/p) + (1/q) = 1$. More precisely,*

(1) *If $g \in \mathcal{O}L^q(D)$, then g induces a bounded linear functional on $\mathcal{O}L^p(D)$ via the integral pairing*

$$\Phi(f) = \int_D f(z)\overline{g(z)} \, dV(z), \quad f \in \mathcal{O}L^p(D). \tag{1.1}$$

(2) *If $\Phi \in \mathcal{O}L^p(D)^*$, then there is a $g \in \mathcal{O}L^q(D)$ such that Φ is of the form* (1.1). *Moreover, the norms $\|g\|_p$ and $\|\Phi\|^*$ are equivalent.*

Fix a defining function $r(z)$ for D. We denote by ∇_r the complex normal derivative $\nabla_r = \sum_{j=1}^2 (\partial r/\partial \bar{z}_j)(\partial/\partial z_j)$. The Bloch space $\mathcal{B}(D)$ is the set of functions holomorphic on D satisfying $\|f\|_{\mathcal{B}} = \sup\{|r(z)\nabla_r f(z)| : z \in D\} < \infty$. We also prove the following

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