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THE SUBSPACE PROBLEM FOR WEIGHTED INDUCTIVE LIMITS REVISITED

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ABSTRACT. We construct a countable inductive limit of weighted Banach spaces of holomorphic functions on an open subset of $\mathbf{C}^2$ which has a topology that cannot be described canonically by weighted sup-seminorms but such that the sequence of weights is regularly decreasing in the sense of Bierstedt, Meise and Summers. This solves an open problem of these authors from 1986.

1. Introduction. The problem of the projective description of weighted inductive limits of spaces of continuous or holomorphic functions and its applications has been considered by several authors since the article [**5**]. See more references in [**10**] or in the recent article [**2**]. In [**5**] it was proved that a weighted inductive limit of Banach spaces of holomorphic functions defined on an open set G in $\mathbf{C}^{\mathbf{N}}$ admits a canonical projective description by weighted sup-seminorms if the linking maps between the generating Banach spaces are compact. The first counterexample to the problem of projective description of weighted inductive limits of spaces of holomorphic functions was given by the authors in [**10**]. A more natural example for spaces of entire functions was given later in [**9**]. In all the examples known so far the sequence of weights $\mathcal{V} = (v_k)_k$, which define the Banach steps, is not regularly decreasing in the sense of Bierstedt, Meise and Summers [**5**]. This important condition was introduced as an extension of the condition (S) which is sufficient for the projective description. The regularly decreasing condition was characterized in several ways in terms of the corresponding weighted inductive limits of spaces of continuous functions, and it is the condition which characterizes when a Köthe echelon space of order 1 is quasinormable. In particular, if the sequence $\mathcal{V}$ is regularly decreasing the weighted inductive limit $\mathcal{V}H(G)$ and its projective hull $H\overline{V}(G)$, which have the same bounded sets,

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