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AREA INTEGRAL CHARACTERIZATION OF $\mathcal{M}$-HARMONIC HARDY SPACES ON THE UNIT BALL

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ABSTRACT. Characterizations of $\mathcal{M}$-harmonic Hardy spaces $\mathcal{H}^p$ on the unit ball in C^n, $n \geq 1$, in terms of area functions involving gradient and invariant gradient are proved.

1. Introduction. Let B denote the unit ball in C^n, $n \geq 1$, and m the $2n$-dimensional Lebesgue measure on B normalized so that $m(B) = 1$, while σ is the normalized surface measure on its boundary S. For the most part we will follow the notation and terminology of Rudin [**7**]. If $\alpha > 1$ and $\xi \in S$ the corresponding Koranyi approach region is defined by

$$D_\alpha(\xi) = \{z \in B : |1 - \langle z, \xi\rangle| < (\alpha/2)(1 - |z|^2)\}.$$

For any function f on B we define a scale of maximal functions by

$$M_\alpha f(\xi) = \sup\{|f(z)| : z \in D_\alpha(\xi)\}.$$

Let $\tilde{\Delta}$ be the invariant Laplacian on B. That is,

$$(\tilde{\Delta} f)(z) = \frac{1}{n+1}\Delta(f \circ \phi_z)(0), \quad f \in C^2(B),$$

where Δ is the ordinary Laplacian and ϕ_z the standard automorphism of B taking 0 to z, see [**7**]. A function f defined on B is $\mathcal{M}$-harmonic, $f \in \mathcal{M}$, if $\tilde{\Delta} f = 0$.

For $0 < p < \infty$, $\mathcal{M}$-harmonic Hardy space $\mathcal{H}^p$ is defined to be the space of all functions $f \in \mathcal{M}$ such that $M_\alpha f \in L^p(\sigma)$ for some $\alpha > 1$. We note that the definition is independent of α.

For $f \in C^1(B)$, $Df = (\partial f/\partial z_1, \dots, \partial f/\partial z_n)$ denotes the complex gradient of f, $\nabla f = (\partial f/\partial x_1, \dots, \partial f/\partial x_{2n})$, $z_k = x_{2k-1} + ix_{2k}$, $k = 1, \dots, n$, denotes the real gradient of f.

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