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CONTINUOUS MAPS ON IDEAL SPACES OF C^*-ALGEBRAS

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ABSTRACT. The set of ideals of a C^*-algebra can be given a natural topology, which restricts to the hull-kernel topology on the primitive ideals. Our primary interest is to study continuous maps on the space of all ideals, rather than on the subset of primitive ideals. We show how the properties of a map between primitive ideal spaces carry over to properties of the extended map between their ideal spaces.

As an application of these results we determine a number of properties of maps between ideal spaces of tensor products, both minimal and maximal. For example, for C^*-algebras A and B, the map $(\ker\pi, \ker\eta) \mapsto \ker(\pi\otimes\eta) : \mathrm{Id}(A)\times\mathrm{Id}(B) \to \mathrm{Id}(A\otimes B)$ is a homeomorphism onto its range. Finally, we apply these results to tensor products of continuous C^*-bundles.

Introduction. The set of ideals of a C^*-algebra can be given a natural topology associated to the partial ordering given by containment. This topology restricts to the usual hull-kernel topology on the subset of primitive ideals. It is clear that, given a continuous map between the ideal spaces of two C^*-algebras, it need not restrict to a map between primitive ideal spaces. On the other hand, given a continuous map between primitive ideal spaces, does it extend to a continuous map between ideal spaces? A less likely question to ask perhaps is, if we begin with a continuous map from the Cartesian product of two primitive ideal spaces into a primitive ideal space of a third C^*-algebra, will this property be retained when we attempt to extend to a map on the Cartesian product of the ideal spaces? We begin by answering these questions in the affirmative (Section 1).

Given C^*-algebras A and B, we apply these results of Section 1 to the study of three particular maps between the ideal spaces of A and $A\otimes B$, for both minimal and maximal tensor products. Every ideal I generates an ideal $\mathrm{Ext}(I)$ in $A\otimes B$. Every representation of $A\otimes B$

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