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CHARACTERISTIC PAIRS ALONG THE RESOLUTION SEQUENCE

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ABSTRACT. Suppose that f is irreducible in a power series ring in two variables over an algebraically closed field k of characteristic 0. The characteristic pairs of f can be defined from a fractional power series expansion of a solution of f. The singularity of f can be resolved by a finite number of blow ups of points. This subject, which can be traced back to Newton, has been studied extensively. A few references are Abhyankar [**1**], Brieskorn and Knörrer [**2**], Campillo [**3**], Enriques and Chisini [**4**] and Zariski [**7**].

In Sections 1 and 2 we give an exposition of the basic results in the theory of Puiseux series. In Section 3 we give a formula for the characteristic pairs of the transform of f along the sequence of blow ups of points resolving the singularity. As a corollary, we obtain the classical theorem of Enriques and Chisini relating the multiplicity sequence of a resolution and the characteristic pairs of f, and we recover the classical result that the characteristic pairs are an invariant of f. We use an inversion formula of Abhyankar to obtain the results of this paper.

1. The Puiseux series. Let R be a power series ring in two variables over an algebraically closed field k. Then we have the following well-known theorem (see [**2**, pp. 405–406], [**7**, p. 7]).

Theorem 1.1. *Suppose that $f \in R$ is irreducible and (x, y) are regular parameters for R such that the multiplicity $\nu(f) = \nu(f(0, y))$. Then a fractional power series exists* (*called a* Puiseux series) *of y in terms of x. The expansion has the form*

$$y = \sum_{i=1}^{l_1} \alpha_{1,i}\, x^i + b_1\, x^{n_1/m_1}$$

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