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WEAK UNITS IN EPICOMPLETIONS OF ARCHIMEDEAN LATTICE-ORDERED GROUPS

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ABSTRACT. Let κ denote an infinite cardinal number or the symbol ∞. In [**14**] it is shown that in the category of archimedean lattice-ordered groups with l-group homomorphisms, each G has an epicompletion, $\beta^{\kappa}_{\mathcal{A}}(G)$, in which G is κ-completely embedded and which lifts all κ-complete morphisms out of G to epicomplete objects. Since, in general, these epicomplete objects have no concrete realization and since epicomplete objects in the category of archimedean lattice-ordered groups with distinguished weak order unit and unit preserving l-group homomorphisms do have concrete realizations [**4**], it is natural to ask when $\beta^{\kappa}_{\mathcal{A}}(G)$ has a weak unit. In this paper we show that $\beta^{\kappa}_{\mathcal{A}}(G)$ has a weak unit precisely when there is countable $A \subseteq G$ so that the κ-ideal generated by A in G is all of G. Moreover, we show that $\beta^{\omega_0}_{\mathcal{A}}(G)$ has weak unit precisely when every epicompletion of G has weak unit, and we construct an archimedean l-group G with weak unit for which $\beta^{\kappa}_{\mathcal{A}}(G)$ has a weak unit for each κ, $\omega_1 \leq \kappa \leq \infty$, but $\beta^{\omega_0}_{\mathcal{A}}(G)$ has no weak unit.

1. Introduction. Let Arch denote the category of archimedean lattice-ordered groups (l-groups) with l-homomorphisms (that is, homomorphisms which preserve both the group and lattice structure). We note that Arch is closed under products and subobjects. All terms regarding l-groups are standard and can be found in [**8**]. Moreover, some general references for l-groups are [**8**], [**18**], [**2**] and [**12**].

Let A be a subset of an l-group G. The *l-ideal generated by A in G* is the smallest l-ideal of G which contains A and is denoted by (A) or $(A)_G$, and $(\{a\})$ is abbreviated to (a) or $(a)_G$. The set $\{g \in G : |g| \wedge |a| = 0 \text{ for all } a \in A\}$ is denoted by $A^{\perp G}$ or $A^{\perp}$, if the context is clear, and is called the *polar* of A. If A consists of one element a, then one writes $a^{\perp G}$ or $a^{\perp}$ for $\{a\}^{\perp}$. Furthermore, an element $0 \leq u \in G$ is called a *weak unit* if $u^{\perp G} = (0)$.

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