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# DENSE SUBGROUPS AND DIVISIBLE QUOTIENT GROUPS OF LOCALLY COMPACT ABELIAN GROUPS

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**Introduction.** Recall that an Abelian group $A$ is said to be divisible if $nA = A$ for all natural numbers $n$. It follows that homomorphic images of divisible groups are again divisible. A topological group is called monothetic if it contains a dense cycle subgroup. It is known that a locally compact monothetic group is either topologically isomoprhic with the discrete group of integers or is compact (for more details see [**1**, p. 25 and Section 5.5]). The following results concerning a compact monothetic group $G$ were proved as part of Lemma 3 of [**6**] and relate the notions of (topological) density and (algebraic) divisibility: (1) if $H$ is a dense subgroup of $G$, then $G/H$ is divisible, (2) if $G$ is also totally disconnected and $H$ is a subgroup such that $g/H$ is divisible, then $H$ is dense in $G$. These results suggest the following three properties that an LCA, i.e., locally compact Hausdorff Abelian, group $G$ may or may not possess.

**Property $D_1$.** For an arbitrary subgroup $H$, $G/H$ is divisible implies that $H$ is dense in $G$.

**Property $D_2$.** For an arbitrary subgroup $H$, $H$ is dense in $G$ implies that $G/H$ is divisible.

**Property $D_3$.** An arbitrary subgroup $H$ is dense in $G$ if and only if $G/H$ is divisible.

In this note we study the beautiful interplay between the topological property of denseness of subgroups and the algebraic property of divisibility of quotients in the form of the three properties defined above. Additionally, we shall say that an LCA group $G$ has property

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