

THE RESTRICTED TANGENT BUNDLE OF SMOOTH CURVES IN GRASSMANNIANS AND CURVES IN FLAG VARIETIES

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ABSTRACT. Let X be a smooth curve of genus $g \geq 2$ over an algebraically closed base field k of any characteristic. Denote by $G(r, \nu)$ the Grassmannian of the rank r quotients of k^ν and by \mathcal{Q} the universal quotient bundle of $G(r, \nu)$. Let us consider degree d embeddings $\varphi : X \rightarrow G(r, \nu)$. We prove that, for $d \geq \nu + r(g - 1)$ and $(\nu, r, d) \neq (4, 2, 2g + 2)$, varying φ we obtain as restricted quotient bundles $\varphi^*(\mathcal{Q})$ points of an open dense subset of the moduli space $M(X; r, d)$ of rank r stable vector bundles on X with degree d . We can extend this result to the flag varieties. For the projective spaces \mathbf{P}^n , we obtain that if d is large with respect to g , $d \geq ng + 1$, then degree d embeddings $\varphi : X \rightarrow \mathbf{P}^n$ cover a dense open subset of the moduli space $M(X; n, (n + 1)d)$ by means of the restricted tangent bundles $\varphi^*(T_{\mathbf{P}^n})$. This fact does not hold for restricted tangent bundles of a Grassmannian $G(r, \nu)$ with $2 \leq r \leq \nu - 2$. However, for a large degree d , we are able to characterize the restricted tangent bundles $\varphi^*(T_{G(r, \nu)})$ of a Grassmannian, obtaining that in general they are stable. For an elliptic curve Y , we show that in characteristic 0 there is a degree d embedding of Y in a Grassmannian with a stable restricted tangent bundle if and only if there is not a numerical restriction to its existence.

Introduction. Let X be a smooth curve of genus $g \geq 2$ over an algebraically closed base field k of any characteristic. Let $M(X; r, d)$ be the irreducible smooth variety parameterizing the stable vector bundles on X with rank $r > 0$ and degree d . We have $\dim M(X; r, d) = r^2(g - 1) + 1$. If L is a line bundle on X with degree d , we denote by $M(X; r, L)$ the irreducible smooth subvariety of $M(X; r, d)$ parameterizing the stable rank r vector bundles with determinant L . We have $\dim M(X; r, L) = (r^2 - 1)(g - 1)$.

Let us consider degree d embeddings $\varphi : X \rightarrow \mathbf{P}^n$, $n \geq 3$. Several authors studied the semi-stability and stability of the restricted tangent

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