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ENFORCED DRAINAGE TERRAIN MODELS USING MINIMUM NORM NETWORKS AND SMOOTHING SPLINES

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ABSTRACT. Some techniques to overcome the problem of enforced drainage in mathematical terrain modeling are presented. Data available about the terrain consist of a set of scattered benchmarks and an idealized, piecewise linear hydrographic net. First, a cubic spline minimum norm network, MNN, on a triangulation of the benchmark data is created, in order to create a first impression of the terrain. The cubics from the MNN supply temporary profiles along the edges of the triangulation; these are modified on those edges which intersect the hydrographic net in order to simulate erosion of the terrain. This is done by weighted smoothing spline techniques. A network of monotonic cubic arcs is created on the edges of the hydrographic net. To arrive at the final surface model, a blending method is applied that requires the specification of elevations and gradient vectors along all edges. Although the elevations along the edges that lie in the hydrographic net are monotonic by construction, the gradient there should also be parallel to the direction of the hydrographic net. This is achieved approximately in an L^2-sense. The result is a differentiable surface which interpolates the stream channels as well as the modified spline MNN.

1. Introduction. An important problem in mathematical terrain modeling is that of enforced drainage. Terrain models based on topographic benchmark data often suffer from undulations and this results in "dams" and associated pools forming in what should be stream channels. In part, this is the result of not constraining the terrain model to honor the hydrographic net. We will develop some techniques that help to overcome this problem. Our approach was motivated by aerial observation of some prairie terrain in western Canada. It was strikingly evident that an underlying, slowly undulating surface was eroded by systems of streams, and that a terrain model based solely on point data

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