ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 3, Fall 2000

MONOTONICITY AND ROTUNDITY PROPERTIES IN BANACH LATTICES

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ABSTRACT. Some general results on geometry of Banach lattices are given. It is shown among others that uniform rotundity or rotundity coincide to uniform or strict monotonicity, respectively, on order intervals in positive cones of Banach lattices. Several equivalent conditions on uniform and strict monotonicity are also discussed. In particular, it is proved that in Banach function lattices uniform and strict monotonicity may be equivalently defined on orthogonal elements. It is then applied to show that p-convexification $E^{(p)}$ of E is uniformly monotone if and only if E possesses that property. A characterization of local uniform rotundity of Calderón-Lozanovskii spaces is also presented.

Introduction. In the following $\mathbf{N}$, $\mathbf{R}$ and $\mathbf{R}_+$ stand for the sets of natural numbers, reals and nonnegative reals, respectively. The triple (T, Σ, μ) stands for a nonatomic, complete and σ-finite measure space. By $L^0 = L^0(\mu)$ we denote the space of all (equivalence classes of) Σ-measurable functions x from T to $\mathbf{R}$. By $E = (E, \leq, \| \, \|)$ we denote an abstract *Banach lattice* with a partial order $\leq$ (see [**2**], [**19**]) as well as a *Banach function space*, being a Banach sublattice of L^0 such that

(i) If $x \in L^0$, $y \in E$ and $|x| \leq |y|$, μ almost everywhere, then $x \in E$ and $\|x\| \leq \|y\|$.

(ii) There exists $x \in E$ such that $x(t) \neq 0$ for all $t \in T$.

The positive cone of E will be denoted by E_+. In the case of the counting measure space $(\mathbf{N}, 2^{\mathbf{N}}, \mu)$ where $\mu(A) = \operatorname{Card}(A)$ for every $A \subset \mathbf{N}$, a Banach function space E is called a *Banach sequence space.*

As usual, for every $x \in L^0$, $\operatorname{supp} x = \{t \in T : x(t) \neq 0\}$ is the support of x and χ_A is a characteristic function of $A \in \Sigma$. We denote by $S(E)$ and $B(E)$ the unit sphere and the unit ball in E, respectively. A Banach lattice E is said to be *order continuous* if, for every nonincreasing

Received by the editors on June 6, 1998, and in revised form on August 17, 1999.
Research of the first author supported by KBN grant 2 P03A 03110.
Research of the third author supported by KBN grant 2 P03A 05009.