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JACOBI FORMS AND GENERALIZED RC-ALGEBRAS

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ABSTRACT. Using the recently found Rankin-Cohen type brackets on the spaces of Jacobi forms, we define generalized Rankin-Cohen algebras. We study their algebraic properties and give examples generalizing the elliptic cases.

1. Introduction. Classically, there are many interesting connections between differential operators and the theory of elliptic modular forms, and several interesting results have been obtained (see [**6**], [**8**], for instance). In 1956, Rankin gave a general description of the differential operators which sent modular forms to modular forms [**6**]. Later, Cohen constructed certain differential bilinear operators acting on the graded ring $M_*(\Gamma)$ of modular forms on the group $\Gamma \subset PSL(2, Z)$ and used them to construct modular forms with interesting Fourier coefficients [**4**]. In 1990, Zagier studied the algebraic properties of these bilinear operators and called them Rankin-Cohen brackets [**8**]. Moreover, the Rankin-Cohen brackets are shown to appear as the various terms in the (convergent) expansion of the composition of two symbols in a certain symbolic calculus associated with $SL(2, \mathbf{R})$ [**7**]. The existence of infinitely many identities among the Rankin-Cohen brackets motivated the definition of Rankin-Cohen algebras whose properties have been studied in detail in [**8**].

Recently, the theory of Jacobi forms has been studied extensively and systematically, first by Eichler and Zagier [**5**] and many others. It turns out that Jacobi forms are connected with modular forms of half-integral weight as well as integral weight, Siegel modular forms and elliptic curves. It was shown that the heat operator plays an important role connecting Jacobi forms and elliptic modular forms. In [**1**], [**2**], [**3**] the generalization of the Rankin-Cohen brackets, which involves the heat operator, to Jacobi forms has been found.

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