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THE EQUIVARIANT CATEGORY OF PROPER G-SPACES

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Introduction. If G is a Lie group, then by a G-space we mean a completely regular space X together with a fixed action of G on X. We follow the standard notation of the theory of transformation groups used in [**5**] and [**9**]. In this paper we deal with "minimax invariants" of a G-space. If one restricts consideration to compact Lie groups, then a substantial general theory of G-genus, G-index, as well as G-category in the sense of Lusternik-Schnirelmann has already been developed. See [**1**], [**11**] and [**17**]. In contrast, if G is not compact, results on such invariants become scarce in the literature. The aim of this paper is to give a general overview of the invariants of type Lusternik-Schnirelmann for an interesting class of G-spaces without the assumption of compactness for the group G; that is, G-spaces with proper actions. The crucial result that allows such a generalization is due to Palais [**19**]. Namely, Palais shows that "slices" still exist for proper G-spaces. This fact leads us to extend a remarkable amount of the theory of G-spaces with G compact to G-spaces with proper actions.

With this paper we intend to point out that there is no real difficulty in extending to proper actions the notion of equivariant Lusternik-Schnirelmann category defined for compact transformation groups in [**11**] and [**17**]. In fact, we show that the basic properties of equivariant "minimax" invariants still hold for proper G-spaces and more generally for Cartan G-spaces.[1] See Section 1 for definitions. Moreover, in many cases the computation of the equivariant category of a proper G-space can be reduced to the compact case, see (2.6) below. The main result of this paper states a Kranosielski type theorem for proper actions of discrete groups on acyclic manifolds (Theorem 4.2). Incidentally, we give in addition an alternative proof of a Kranosielski type theorem for (co)homology spheres due to Marzantowicz (Theorem 4.4). Finally we

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