IDEALS GENERATED BY PROJECTIONS
AND INDUCTIVE LIMIT C^*-ALGEBRAS

CORNEL PASNICU

ABSTRACT. We introduce two classes of inductive limit C^*-algebras which generalize the AH algebras: the GAH algebras (GAH stands for "generalized AH") and a subclass of it, the *strong GAH* algebras. We give necessary and sufficient conditions for an ideal of a GAH algebra to be generated by projections which, in particular, gives necessary and sufficient conditions for a GAH algebra to have the ideal property, i.e., any ideal is generated by projections. We prove that if $0 \to I \to A \to B \to 0$ is an exact sequence of C^*-algebras such that A is a GAH algebra, then A has the ideal property if and only if I and B have the ideal property. We describe the lattice of ideals generated by projections of a strong GAH algebra and also the partially ordered set of the stably cofinite ideals generated by projections of a strong GAH algebra A under the additional assumption that the projections in $M_\infty(A)$ satisfy the Riesz decomposition property. These results generalize some of our previous theorems involving AH algebras.

1. Introduction. A C^*-algebra has the ideal property if any ideal is generated, as an ideal, by projections ([**13**]). In this paper, by "ideal" we shall mean "closed, two-sided ideal". An AH algebra is the inductive limit of a sequence of C^*-algebras which are finite direct sums of C^*-algebras of the form $PC(X, M_n)P$, where X is a connected, finite CW complex and P is a projection in $C(X, M_n)$ ([**1**]). The AH algebras with the ideal property present interest since they include two important classes of C^*-algebras: the *simple AH* algebras and the *real rank zero AH* algebras ([**4**]), about which a lot of interesting results have been proved in the last years. The study of the AH algebras with the ideal property is related to a problem of Effros ([**6**]) (namely, find suitable topological invariants for AH algebras), and also to Elliott's project on the classification of the separable, amenable C^*-algebras by invariants including K-theory ([**7**]). The AH algebras with the ideal property have been studied in [**13**], [**9**], [**10**], [**11**] and [**12**].