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ON THE SECOND FUNDAMENTAL TENSOR OF REAL HYPERSURFACES IN QUATERNIONIC HYPERBOLIC SPACE

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ABSTRACT. We study real hypersurfaces with constant quaternionic sectional curvature in the quaternionic hyperbolic space and the action of the curvature operator on the Weingarten endomorphism. We also introduce examples of ruled real hypersurfaces.

1. Introduction. A quaternionic Kaehlerian manifold is called a quaternionic space form if it is connected, simply connected and it is endowed with a complete metric g of constant quaternionic sectional curvature c. The study of real hypersurfaces when $c > 0$ is rather developed (cf. [**1**], [**7**], [**9**]). Besides, these authors deal with both $c < 0$ and $c > 0$. Our purpose is to study real hypersurfaces in quaternionic hyperbolic space ($c < 0$) of constant quaternionic sectional curvature $c = -4$, $\mathbf{Q}H^m$, $m \geq 2$, by paying attention to the second fundamental tensor.

We describe in Section 2 a semi-Riemannian manifold of index 3 that is a semi-Riemannian submersion (cf. [**5**]) over $\mathbf{Q}H^m$ with time-like totally geodesic fibers as well as a principal fiber bundle over $\mathbf{Q}H^m$ with structural group $\mathbf{S}^3$.

A real hypersurface M of a quaternionic space form is said to be ruled if its maximal quaternionic distribution $\mathbf{D}$ of the tangent bundle of M, TM, is integrable. This condition is equivalent to $g(AX, Y) = 0$ for any X, Y in $\mathbf{D}$, where A is the Weingarten endomorphism of M (cf. [**8**]). In Section 4, we construct a family of ruled real hypersurfaces in $\mathbf{Q}H^m$, $m \geq 2$, which proves that the class of such real hypersurfaces is not empty, so Theorem 2 is meaningful.

$\mathbf{D}$ is a linear subbundle of TM which inherits two different metric tensors. The first one is the simple restriction of g, so it seems natural to keep the same name. The second one comes from the

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