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STURM-LIOUVILLE EIGENVALUE PROBLEMS FOR HALF-LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we discuss the half-linear Sturm-Liouville eigenvalue problem

$$\begin{cases} (p(t)|x'|^{\alpha-1}x')' + \lambda q(t)|x|^{\alpha-1}x = 0, & a \le t \le b, \\ Ax(a) - A'x'(a) = 0, & Bx(b) + B'x'(b) = 0, \end{cases}$$

for the case where $q(t)$ may change signs in the interval $[a,b]$. As a typical result we have the following theorem. If $q(t)$ takes both a positive value and a negative value, then the totality of eigenvalues consists of two sequences $\{\lambda_n^+\}_{n=0}^\infty$ and $\{\lambda_n^-\}_{n=0}^\infty$ such that $\cdots < \lambda_n^- < \cdots < \lambda_1^- < \lambda_0^- < 0 < \lambda_0^+ < \lambda_1^+ < \cdots < \lambda_n^+ < \cdots$, $\lim_{n\to\infty} \lambda_n^+ = +\infty$ and $\lim_{n\to\infty} \lambda_n^- = -\infty$. The eigenfunctions associated with $\lambda = \lambda_n^+$ and λ_n^- have exactly n zeros in (a,b). This gives a complete generalization of the well-known results for the linear case ($\alpha = 1$).

1. Introduction. In this paper the second order half-linear ordinary differential equation

$$(1.1) \qquad (p(t)|x'|^{\alpha-1}x')' + \lambda q(t)|x|^{\alpha-1}\, x = 0, \quad a \le t \le b,$$

is considered together with the boundary conditions

$$(1.2) \qquad Ax(a) - A'x'(a) = 0, \qquad Bx(b) + B'x'(b) = 0.$$

In equation (1.1) we assume that $\alpha > 0$ is a positive constant, p and q are real-valued continuous functions for $a \le t \le b$, and $p(t) > 0$, $a \le t \le b$, and $\lambda \in \mathbf{R}$ is a real parameter. In the boundary conditions (1.2), A, A', B and B' are given real numbers such that $A^2 + A'^2 \neq 0$ and $B^2 + B'^2 \neq 0$.

If $\alpha = 1$, then equation (1.1) reduces to the linear equation

$$(1.3) \qquad (p(t)x')' + \lambda q(t)x = 0, \quad a \le t \le b,$$

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