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DECOMPOSING THE DISENTANGLING ALGEBRA ON WIENER SPACE

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ABSTRACT. For any functional H in the disentangling algebra $\mathcal{A}_{a,t}^c$ and any partition $a = r_0 < r_1 < \cdots < r_h = t$ of $[a,t]$, we show that H can be decomposed into a sum of terms where each term is the $*$ product of h functions, one from each of the disentangling algebras $\mathcal{A}_{r_{j-1},r_j}^c$, $j = 1, \dots, h$. Correspondingly, we show that the operator-valued path integral of H, $K_\lambda^{a,t}(H)$, can be written as the sum of time-ordered products of the h operators associated with the h subintervals of $[a,t]$. We also consider various special cases of these results. The $*$ product is a noncommutative multiplication (or concatenation) of functions on Wiener space.

1. Introduction. A family $\{\mathcal{A}_t, t > 0\}$ of commutative Banach algebras of functionals on Wiener space was introduced in [**4**] and it was shown that for every $F \in \mathcal{A}_t$, the functional integral $K_\lambda^t(F)$ exists and is given by a time-ordered perturbation expansion which serves to disentangle, in the sense of Feynman's operational calculus for noncommuting operators, the operator $K_\lambda^t(F)$. The first author and Lapidus introduced in [**6**] the noncommutative operations $*$ and $\dot{+}$ on Wiener functionals, and they showed as one of their main results that if $F \in \mathcal{A}_{t_1}$ and $G \in \mathcal{A}_{t_2}$, then $F * G \in \mathcal{A}_{t_1+t_2}$ and $K_\lambda^{t_1+t_2}(F * G) = K_\lambda^{t_1}(F)K_\lambda^{t_2}(G)$. It follows then that the product of operators which can be disentangled (in their framework) can itself be disentangled.

Our aim in this paper is related but quite different. It is to show that for any $H \in \mathcal{A}_{a,t}^c$ (see Definition 2.4 below) and any partition $a = r_0 < r_1 < \cdots < r_h = t$ of $[a,t]$, H can be decomposed into a sum of terms where each term is the $*$ product of F_j's one from each of the algebras $\mathcal{A}_{r_{j-1},r_j}^c$, $j = 1, \dots, h$. This decomposition of the function H along with the relationship between the functional integrals $K_\lambda^{r_{j-1},r_j}(\cdot)$,

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