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FACTOR MAPS, ENTROPY AND FIBER CARDINALITY FOR MARKOV SHIFTS

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ABSTRACT. It is well known that a factor map between transitive shifts of finite type either preserves entropy and is bounded-to-1 or it does not preserve entropy and is uncountable-to-1. In this paper we elucidate the relation between entropy and fiber cardinality for factor maps between transitive locally compact Markov shifts. We show that every countable-to-1 factor map increases the Gurevic entropy while every finite-to-1 factor map preserves Gurevic entropy. We study finite-to-1 proper factor maps and show that they additionally preserve positive and strongly positive recurrence. Then we investigate finite-to-1 proper factor maps between Markov shifts which have an expansive 1-point compactification. We conclude the paper with some examples showing that properly finite-to-1 and properly countable-to-1 factor maps exist between synchronized systems.

Introduction. Shifts of finite type (SFT), [**19**], [**20**], can be generalized in two ways. One can keep the compactness of the shift space but relax the Markov property which leads to synchronized systems and coded systems, [**1**], [**5**], [**10**], [**13**], [**14**]. Or one can keep the Markov property and relax the compactness which leads to locally compact Markov shifts, [**6**]–[**9**], [**12**]–[**15**], [**19**], [**20**]. Markov shifts and coded systems are strongly related. We quote two results to illuminate this relation. Coded systems are those compact subshifts which are the surjective factors of transitive Markov shifts [**14**], and a subshift compactification of a transitive locally compact Markov shift is always coded [**13**].

In this paper we study factor maps between locally compact Markov shifts and complete the results on factor maps between coded and synchronized systems obtained in [**5**].

A *subshift* is a shift invariant subset S of $\mathbf{N}^{\mathbf{Z}}$, endowed with the product topology of the discrete topology on $\mathbf{N} = \{1, 2, \dots\}$, together

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