

C^* -ALGEBRAS OF DYNAMICAL SYSTEMS OF QUASI ROTATIONS ON TORI

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ABSTRACT. In this note we determine the isomorphism classes of the crossed product C^* -algebras of affine (n, λ) quasi rotations of \mathbf{T}^n .

1. Introduction. There have been considerable contributions to the computation of K -theoretical and isomorphism invariants of C^* -algebras of dynamical systems on the n -torus \mathbf{T}^n , which include certain noncommutative tori [5], [3], [7]. Riedel [5] classified the crossed products of $C(\mathbf{T}^n)$ by minimal rotations of \mathbf{T}^n , i.e., minimal transformations of \mathbf{T}^n with degree matrix $D(\phi) = I_n$. He showed that the set of eigenvalues of ϕ is a complete isomorphism invariant. When ϕ is a minimal homeomorphism of \mathbf{T}^n with quasi discrete spectrum, Packer [3] computed the tracial range of $K_0(C(\mathbf{T}^n) \rtimes_{\alpha_\phi} \mathbf{Z})$. For $n = 2$, Rouhani [7] classified, by using K -theoretical invariants, the isomorphism classes of the crossed product C^* -algebras $C(\mathbf{T}^2) \rtimes_{\alpha_\phi} \mathbf{Z}$, where ϕ is an (affine) irrational quasi rotation of \mathbf{T}^2 . That is an (affine) transformation that has a unitary eigenvalue $\lambda = e^{2\pi i\theta}$ (θ irrational) with a unitary eigenfunction f having degree matrix $D(f) = [n, m] \neq 0$, where n, m are relatively prime and the degree matrix $D(\phi)$ satisfies $\text{rank}_{\mathbf{Q}}(D(\phi) - I_2) = 1$. The concept of quasi rotation admits a natural generalization to an n quasi rotation for transformations $\phi : \mathbf{T}^n \rightarrow \mathbf{T}^n$. Roughly speaking, ϕ is now required to have $n - 1$ eigenvalues while the degree matrix $D(\phi)$ still satisfies $\text{rank}_{\mathbf{Q}}(D(\phi) - I_n) = 1$. (See Definition 2 and Lemma 3.)

Our main result, which generalizes the main theorem in [7] to \mathbf{T}^n , $n \geq 3$, is the characterization, using K -theoretical invariants, of the isomorphism classes of crossed products $C(\mathbf{T}^n) \rtimes_{\alpha_\phi} \mathbf{Z}$ of \mathbf{T}^n , where ϕ is an affine n quasi rotation, provided some additional conditions are

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