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ON SEMI-RYAN COMPLEX SUBMANIFOLDS IN AN INDEFINITE COMPLEX SPACE FORM

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ABSTRACT. The purpose of this paper is to study several classes of an n-dimensional complete space-like complex submanifold of an $(n+p)$-dimensional indefinite complex space form $M^{n+p}_{0+t}(c)$ of index $2t$, and of an n-dimensional space-like complex hypersurface of a complex Minkowski space C^{n+1}_1 in terms of $RS = 0$.

1. Introduction. The theory of indefinite complex submanifolds of an indefinite complex space form is one of the most interesting topics in differential geometry, and it has been investigated by many geometers from various points of view (see [**1**], [**3**], [**6**], [**7**], [**10**] and [**14**]–[**17**], etc.).

Let $M^m_t(c)$ be an m-dimensional semi-definite complex space form of constant holomorphic sectional curvature c and of index $2t$, $0 \leqq t \leqq m$. As is well known, it globally consists of three kinds of complex space forms: the semi-definite complex projective space $CP^m_t(c)$, the semi-definite complex Euclidean space C^m_t, or the semi-definite complex hyperbolic space $CH^m_t(c)$, according to whether $c > 0$, $c = 0$ or $c < 0$.

Let M be a semi-definite Kaehler manifold, and let us denote by R and S the Riemannian curvature tensor and the Ricci tensor on M, respectively. Recently, the present authors [**5**] have given a complete classification of semi-symmetric complex hypersurfaces in a semi-definite complex space form $M^m_1(c)$. Here the notion of semi-symmetric means $R(X,Y)R = 0$ for any vector fields X and Y on M. In this paper we want to introduce another notion of $R(X,Y)S = 0$, which is said to be

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