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A RESTRICTED DICHOTOMY OF EQUIVALENCE CLASSES FOR SOME MEASURES OF DEPENDENCE

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ABSTRACT. In some limit theory for weakly dependent random sequences, a role is implicitly played by certain measures of dependence based on "covariances" of random variables taking their values in Banach spaces. Here it is shown that in a certain restricted sense, there is a "dichotomy of equivalence classes" for measures of dependence of that type that involve "∞-norms" of the random variables. The question of a possible corresponding "unrestricted" dichotomy of equivalence classes remains open.

1. Introduction. In probability theory, there is a large literature on limit theorems under "strong mixing conditions." The formulations of such mixing conditions are based on "measures of dependence" between σ-fields of events. Some of that limit theory involves random variables taking their values in a Hilbert space or (more generally) in a Banach space.

Building on the work of Rosenblatt [**13**, Chapter 7], Dehling and Philipp [**7**] and other researchers, the author, Bryc, and Janson wrote a series of papers [**4**], [**5**], [**6**] on the relationships (e.g., "dominations" or "equivalencies") within certain classes of measures of dependence. The latter paper [**6**] studied in detail a broad class of measures of dependence involving "covariances" of random variables taking their values in general Hilbert spaces or Banach spaces.

For the measures of dependence of that latter type that involve the "∞-norms" of those random variables, it turns out that there is what one might refer to as a "restricted dichotomy of equivalence classes." It will be formulated in Theorem 1.8 and Remark 1.12 below. The question of a possible corresponding "unrestricted dichotomy" remains open; more on that in Remark 1.12.

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