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MORITA EQUIVALENCE OF C^*-CROSSED PRODUCTS BY INVERSE SEMIGROUP ACTIONS AND PARTIAL ACTIONS

NÁNDOR SIEBEN

ABSTRACT. Morita equivalence of twisted inverse semigroup actions and discrete twisted partial actions are introduced. Morita equivalent actions have Morita equivalent crossed products.

1. Introduction. Morita equivalence of group actions on C^*-algebras was studied by Combes [**3**], Echterhoff [**5**], Curto, Muhly and Williams [**4**] and Kaliszewski [**11**]. We adapt this notion for both Busby-Smith and Green type inverse semigroup actions, introduced in [**18**] and [**19**]. We show that Morita equivalence is an equivalence relation and that Morita equivalent actions have Morita equivalent crossed products. The close connection between inverse semigroup actions and partial actions [**18**], [**9**], [**19**] makes it easy to find the notion of Morita equivalence for discrete twisted partial actions. In Section 4 we work out some of the details of discrete twisted partial crossed products, continuing the work started in [**8**]. The fact that Morita equivalent twisted partial actions have Morita equivalent crossed products will then follow from the connection with semigroup actions. In [**1**] Abadie, Eilers and Exel introduced Morita equivalence of crossed products by Hilbert bimodules. We show that this definition is equivalent to our definition of Morita equivalence on the common special case of partial actions by $\mathbf{Z}$.

2. Preliminaries. In this section we discuss isomorphisms of Hilbert bimodules. Note that our Hilbert bimodules are not necessarily full. Our references for Hilbert modules are [**10**], [**12**] and [**16**].

Definition 2.1. The triple (ϕ_A, ϕ, ϕ_B) is called an *isomorphism*

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