ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 2, Summer 2001

DEGENERATE HOMOGENEOUS STRUCTURES OF TYPE $\mathcal{S}_1$ ON PSEUDO-RIEMANNIAN MANIFOLDS

A. MONTESINOS AMILIBIA

ABSTRACT. We obtain all of the pseudo-Riemannian manifolds endowed with homogeneous structures defined by isotropic vector fields. Thus, the (general pseudo-Riemannian) class $\mathcal{S}_1$ of homogeneous structures is fully determined.

1. Introduction. Ambrose and Singer [**1**] gave a characterization for a connected, simply connected and complete Riemannian manifold to be homogeneous, in terms of a (1,2) tensor field S on the manifold. This characterization extends the classical one given by Cartan of Riemannian symmetric spaces as the spaces of parallel curvature, which correspond to Ambrose-Singer's case $S = 0$. That characterization has also permitted Tricerri and Vanhecke [**8**] to classify those homogeneous Riemannian manifolds into eight classes which are defined by the invariant subspaces of certain space $\mathcal{S}_1 \oplus \mathcal{S}_2 \oplus \mathcal{S}_3$. In [**8**] it is proved that a connected, simply connected and complete Riemannian manifold admits a nonvanishing homogeneous structure S of type $\mathcal{S}_1$ if and only if it is isometric to the hyperbolic space.

Gadea and Oubiña [**4**] have extended the characterization in [**1**] to the pseudo-Riemannian case of any signature and proved that a connected, simply connected and complete pseudo-Riemannian manifold admits a homogeneous pseudo-Riemannian structure if and only if it is reductive homogeneous. As is well known, in the Riemannian case every homogeneous manifold is complete and reductive.

Gadea and Oubiña give in [**6**] a classification for the pseudo-Riemannian case of any signature similar to that given in [**8**] for Riemannian homogeneous structures, and they moreover characterize the three primitive classes. From now on we shall focus attention on the first class, $\mathcal{S}_1$. A connected, simply connected and complete pseudo-Riemannian manifold (M, g) of any signature (M, g) admits [**6**] a *nondegenerate*,

1991 AMS *Mathematics Subject Classification.* 53C30, 53C50, 53C15.
Key words and phrases. Homogeneous structures, pseudo-Riemannian manifolds.
Received by the editors on February 11, 1999.