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WEIGHTED VERSION OF MULTIVARIATE OSTROWSKI TYPE INEQUALITIES

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ABSTRACT. We establish two weighted integral identities and use them to prove a number of inequalities of Ostrowski type for functions of several variables. The results in the paper extend some known results of Pečarić and Savić as well as some recent results of Anastassiou.

1. Introduction and preliminary results. The results in this paper are motivated by the following integral inequality that was proved in 1938 by Ostrowski [**12**].

Theorem A. *Let f be a differentiable function on $[a,b]$, and let $|f'(x)| \leq M$ on $[a,b]$. Then, for every $x \in [a,b]$,*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(t)\,dt \right| \leq \left[\frac{1}{4} + \frac{\left(x - ((a+b)/2)\right)^2}{(b-a)^2} \right] (b-a)M. \tag{1.1}$$

Some generalizations of this inequality, obtained by Milovanović [**8, 9**], Milovanović and Pečarić [**10**] and Fink [**4**] were noted in [**11**, pp. 468–471]. Recently, Anastassiou [**1**], [**2**] proved some more general inequalities of this type.

We are interested in generalization of (1.1) for functions of several variables. In 1984 Pečarić and Savić [**14**, pp. 263–264] proved the following result.

Theorem B. *Consider a real linear space X of real valued functions $f : Q \to \mathbf{R}$, where Q is a subset of $\mathbf{R}^m$, $m \in \mathbf{N}$, and assume that $\mathbf{1} \in X$ (here $\mathbf{1}$ denotes the constant function $\mathbf{x} \mapsto \mathbf{1}$, $\mathbf{x} \in Q$). Let $A : X \to \mathbf{R}$*

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