ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 1, Spring 2001

THE SHORTEST ENCLOSURE OF TWO CONNECTED REGIONS IN A CORNER

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ABSTRACT. Fix a sector in the Euclidean plane bounded by two rays emanating from a common point. We investigate arc-length minimizing enclosures of two connected regions in this sector with prescribed areas, where the bounding rays do not contribute to the arc-length. We show that the perimeter minimizing configuration is one of two possible types: two concentric circular arcs, or a truncated standard double bubble.

1. Introduction. In nature, a soap bubble configuration encloses and separates several regions of space having fixed volumes while tending to minimize the total surface area. This observation has inspired mathematicians to search for the optimal such configuration of bubbles. The basic question in the mathematics of soap bubbles is the following: given n positive quantities $v_1, \dots, v_n$, how can one enclose and separate n regions of $\mathbf{R}^3$ having volumes $v_1, \dots, v_n$ with the smallest possible surface area? The sphere is well known to enclose a single region of fixed volume with minimal surface area. When $n = 2$, up until this year, the minimal configuration was known only in the special case that the regions enclosed have equal volumes [**3**] (see Figure 1a). If the regions have different volumes, only very recently has anyone [**4**] managed to eliminate troublesome configurations such as the one pictured in Figure 1b.

We consider the simpler, and more tractable, domain of planar bubbles, where the basic problem is to enclose and separate n (not necessarily connected) regions with given areas and minimal perimeter. Of course, if we wish to enclose a single region, the classical isoperimetric inequality gives that our perimeter minimizing enclosure is a circle. More recently, it has been shown that the shortest way to enclose and separate two regions is to use a standard double bubble consisting of

Received by the editors on November 23, 1998, and in revised form on December 31, 1999.