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A CYCLIC ELEMENT CHARACTERIZATION OF MONOTONE NORMALITY

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ABSTRACT. A subcontinuum g of a locally connected continuum X is a cyclic element of X provided that g is maximal with respect to the property that no point separates it. In an earlier paper, Cornette showed that a locally connected continuum is the continuous image of an arc if and only if each cyclic element of X is the continuous image of an arc. In this paper we prove the analogous theorem for monotonically normal continua by showing that a locally connected continuum X is monotonically normal if and only if each cyclic element of X is monotonically normal.

Definition. A continuum is a compact connected Hausdorff space. A continuum is called an arc provided that it is a nondegenerate ordered continuum.

Notation. If $S \subset X$, $\text{Int}_X(S)$ will denote the interior of S with respect to X or simply $\text{Int}\,(S)$ if the superspace is clear. Similarly, $\partial_X(S)$ or $\partial(S)$ will denote the boundary of S with respect to X.

Definition. A cyclic element C of a locally connected continuum X is a subcontinuum of X that is maximal with respect to the property that no point separates C. If a cyclic element C of X is nondegenerate, C is said to be a true cyclic element of X. A subset A of X is an A-set of X provided that $X - A = \cup G_i$, where each G_i is open in X, $G_i \cap G_j = \varnothing$ for $i \neq j$, $\partial(G_i)$ contains at most one point, and where if C is an open cover of X then all but a finite number of the G_i lie in some element of C. For any two distinct points a and b of X, the intersection of all A-sets in X containing a and b is called the cyclic chain from a to b and is denoted by $C(a, b)$.

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