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# THE OBRECHKOFF TRANSFORM ON SPACES OF GENERALIZED FUNCTIONS

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ABSTRACT. In this paper we study the Obrechkoff transformation on some generalized functional spaces by employing the kernel method. Thus we extend the results of Baier and Glaeske for the Laplace transformation and of Betancor and Rodríguez-Mesa for the $K$ transformation. Furthermore, from the results on the Obrechkoff transformation, the analogous ones for the Krätzel transformation follow as special cases.

**1. Introduction.** In this paper we aim to define and study the so-called Obrechkoff integral transform on some spaces of generalized functions. This transform seems to be one of the most general and effective generalizations of the Laplace transform, related to differential operators of Bessel-type, $m \in \mathbf{N}$, $\beta := m - (\alpha_0 + \alpha_1 + \cdots \alpha_m) > 0$, $\gamma_k = (\alpha_k + \alpha_{k+1} + \cdots + \alpha_m - m + k)/\beta$, $k = 1, \dots, m$,

$$(1.1) \quad B = x^{\alpha_0} \frac{d}{dx} x^{\alpha_1} \frac{d}{dx} \cdots x^{\alpha_{m-1}} \frac{d}{dx} x^{\alpha_m} = x^{-\beta} \prod_{j=1}^{m} \left( x \frac{d}{dx} + \beta \gamma_k \right),$$

also called "hyper-Bessel differential operators." As most simple cases of (1.1), the second order differential operator of Bessel and the $m$th order differentiation $D^m = (d/dx)^m$ appear. Many authors have introduced and studied Laplace-type integral transforms related to very special cases of (1.1). It happened that the most general transform of this kind had been introduced in 1958 by Obrechkoff [**12**], but his results remained unknown for a period of time. Dimovski [**4**] established that this transform can be successfully used for building an operational calculus for the operators (1.1) (and their linear right inverse operators, the "hyper-Bessel" integral operators $L$). In [**4**], [**5**], some basic properties of the Obrechkoff transform are found for

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