ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 31, Number 1, Spring 2001

EQUAL SUMS OF LIKE POWERS

AJAI CHOUDHRY

ABSTRACT. This paper is concerned with four diophantine systems, namely, (i) $\sum_{i=1}^4 x_i^k = \sum_{i=1}^4 y_i^k$, $k = 1,2,4$; (ii) $\sum_{i=1}^4 x_i^k = \sum_{i=1}^4 y_i^k$, $k = 1,3,4$; (iii) $\sum_{i=1}^4 x_i^k = \sum_{i=1}^4 y_i^k$, $k = 2,3,4$; (iv) $\sum_{i=1}^3 x_i^k = \sum_{i=1}^3 y_i^k$, $k = 2,3,4$. Parametric solutions as well as numerical examples of solutions in positive integers of the first three diophantine systems have been obtained in the paper. For the fourth diophantine system, solutions do not exist in positive real numbers and a single numerical solution in integers has been obtained.

This paper is concerned with the following four diophantine systems relating to the problem of equal sums of like powers:

$$\begin{array}{lll} \text{I.} & x_1^k + x_2^k + x_3^k + x_4^k = y_1^k + y_2^k + y_3^k + y_4^k, & k = 1,2,4. \\ \text{II.} & x_1^k + x_2^k + x_3^k + x_4^k = y_1^k + y_2^k + y_3^k + y_4^k, & k = 1,3,4. \\ \text{III.} & x_1^k + x_2^k + x_3^k + x_4^k = y_1^k + y_2^k + y_3^k + y_4^k, & k = 2,3,4. \\ \text{IV.} & x_1^k + x_2^k + x_3^k = y_1^k + y_2^k + y_3^k, & k = 2,3,4. \end{array}$$

Solutions of these diophantine systems have not been published before. We will obtain two parametric solutions of the diophantine system I, one parametric solution of system II and one parametric solution of system III. We also give an additional method of generating infinitely many integer solutions of system III. As numerical examples we will obtain solutions in positive integers of these three diophantine systems. Finally, we obtain the following numerical solution of system IV:

$$\begin{aligned} &358^2 + (-815)^2 + 1224^2 = (-410)^2 + (-776)^2 + 1233^2, \\ &358^3 + (-815)^3 + 1224^3 = (-410)^3 + (-776)^3 + 1233^3, \\ &358^4 + (-815)^4 + 1224^4 = (-410)^4 + (-776)^4 + 1233^4. \end{aligned} \tag{1}$$

This solution is particularly interesting since it has been proved earlier by Palama [**4**] that the diophantine system IV has no solutions in

Received by the editors on April 5, 1999, and in revised form on September 20, 1999.