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SYSTEMS OF INTEGRAL EQUATIONS ON THE HALF-AXIS WITH QUASI-DIFFERENCE KERNELS

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ABSTRACT. A system of integral equations on the half-axis with a quasi-difference kernel in matrix form

$$(1)\qquad Y(x) = \int_0^\infty K_1(x-t)Y(t)\,dt + \int_0^\infty K_2(x+t)Y(t)\,dt + U(x), \quad x > 0$$

is considered under the assumptions $e^{s|x|}K_1(x) \in L(-\infty,\infty)$, $e^{sx}K_2(x) \in L(0,\infty)$ and $e^{-sx}U(x) \in L(0,\infty)$, with solutions satisfying the condition $e^{-sx}Y(x) \in L(0,\infty)$ where $s \geq 0$.

We show that the problem can be reduced to one or more problems of the type:

$$(2)\qquad \psi(x) = T\psi(x) + \phi(x)$$

with a compact operator T and a vector function $\phi(x)$ such that $e^{sx}\phi(x) \in L(0,\infty)$. The unknown function $\psi(x)$ satisfies the same condition: $e^{sx}\psi(x) \in L(0,\infty)$.

The compactness of T allows us to apply Riesz-Banach theory of linear equations with such operators and establish a Fredholm alternative for (1) and (2).

We will also find some additional conditions under which T is a contraction, so that equation (2) has one and only one solution which can be found by an iteration.

1. Introduction. System (1) is a natural generalization of a system of integral equations on the half-axis with difference kernels, that is,

$$(3)\qquad Y(x) = \int_0^\infty K(x-t)Y(t)\,dt + U(x), \quad 0 < x < \infty.$$

As will be shown in this paper, solutions of (1) and (3) have similar properties although they were found by different methods. System (3) enjoyed a lot of attention over the years and has a rich history. It begins with Wiener and Hopf [**57**] who considered a scalar case and found exact solutions of (3) introducing an algorithm which they

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