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A NOTE ON SOLUTIONS IN $L^1[0,1]$ TO HAMMERSTEIN INTEGRAL EQUATIONS

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ABSTRACT. In this paper we seek solutions in $L^1[0,1]$ to Hammerstein integral equations. Some general existence principles are derived.

1. Introduction. We present some existence principles for the Hammerstein integral equation

$$y(t) = g(t) + \int_0^1 k(t,s) f(s, y(s))\, ds \quad \text{a.e. } t \in [0,1]. \tag{1.1}$$

Throughout we have $k : [0,1] \times [0,1] \to \mathbf{R}$ and $f : [0,1] \times \mathbf{R} \to \mathbf{R}$. In this paper we are mostly interested in solutions which lie in $L^1[0,1]$ (we will for completeness also discuss the case $L^p[0,1]$, $1 < p < \infty$). Banas [**2, 3**] and Emmanuele [**8, 9**] have examined this type of problem extensively over the last ten years or so. Their analyses rely on the notion of measures of weak noncompactness and on the Schauder fixed point theorem. However, in this paper we will use old compactness results of Riesz and Komogorov to establish some very general existence principles (and theory) for (1.1). Our proofs are elementary and follow classical type arguments. It is worth remarking here also that essentially the same reasoning would establish existence principles for Volterra and even Urysohn integral equations.

To conclude the introduction, we gather together some results which will be used frequently in Section 2. We first state the compactness criteria of Riesz and Kolmogorov, see [**1, 4, 5, 7, 11**].

Theorem 1.1 (Riesz). *Let* $\Omega \subseteq L^p[0,1]$, $1 \le p < \infty$. *If*

(i) Ω *is bounded in* $L^p[0,1]$,

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