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SYMM'S LOG KERNEL INTEGRAL OPERATORS

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ABSTRACT. The Bochner integral is applied to prove the compactness of Symm's log kernel integral operators on $\mathcal{L}^1, \mathcal{L}^\infty$ and weighted $\mathcal{L}^p$ spaces when $1 < p < \infty$. Moreover, the ranges of these operators on weighted $\mathcal{L}^p$ spaces are determined, and this is applied to solve singular integral equations.

1. Introduction. The integral equation

$$\pi^{-1} \int_{-1}^{1} f(s) \ln|t-s|\, ds = g(t), \quad t \in]-1, 1[, \tag{1.1}$$

for a given function g is called *Symm's integral equation* by I.H. Sloan and E.P. Stephan [**15**], who named it after G.T. Symm [**16**]. This integral equation arises in many areas in analysis and has applications to potential theory and scattering theory (see [**4, 16**], for example, and the references therein). In [**4**, Section 3], the integral equation (1.1) is solved when g is suitably smooth. In the present paper the equation (1.1) is considered from the viewpoint of operator theory, as in [**7**, Section 13].

To be more precise, let λ denote the Lebesgue measure in the open interval $]-1, 1[$. Symm's log kernel integral operator L_p on the Banach space $\mathcal{L}^p(\lambda)$ is defined by

$$L_p(f)(t) = \pi^{-1} \int_{-1}^{1} f(s) \ln|t-s|\, ds, \quad t \in]-1, 1[, \tag{1.2}$$

for each $f \in \mathcal{L}^p(\lambda)$ whenever $1 \le p \le \infty$.

According to [**7**, Section 13], the operator L_p is compact, whenever $1 < p < \infty$, the proof of which is based upon the Hille-Tamarkin theorem. Furthermore, the range of L_p in the case in which $1 < p < 2$ has been described there, but note that the description in [**7**, Section

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