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A QUADRATURE METHOD FOR THE HYPERSINGULAR INTEGRAL EQUATION ON AN INTERVAL

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ABSTRACT. This paper is concerned with a quadrature method for the approximate solution of the hypersingular integral equation

$$\text{p.f.} \int_0^1 \frac{u(\tau)}{\mid \tau - t \mid^2}\, d\tau = f(t), \qquad 0 \le t \le 1.$$

Stability and error estimates are proved. Numerical experiments are presented which confirm the theoretical estimates.

1. Introduction. In this paper we consider the hypersingular integral equation on the interval

$$(1) \qquad (Du)(t) := \text{p.f.} \int_0^1 \frac{u(\tau)}{\mid \tau - t \mid^2}\, d\tau = f(t), \qquad 0 \le t \le 1,$$

where f is a given function and u is to be found. The integral in (1) is to be interpreted as a Hadamard finite part integral. For the definition of such a finite part integral we refer, e.g., to [**11**, Section 3.2].

Notice that it is possible to consider the hypersingular integral equation of the form (1), disturbed by an integral operator with a smooth kernel function or the hypersingular integral equation on a smooth open curve by using the same methods as in the present paper.

The hypersingular integral equation (1) results from a certain boundary integral method which has attracted the attention of several mathematicians in recent years. In particular, we mention the paper [**3**] of Costabel and Stephan, where the Galerkin method for the hypersingular integral equation on polygons is studied, and the article [**2**] of Costabel, which gives a survey of several boundary integral operators on Lipschitz domains and investigates the Galerkin method for these. In the paper [**19**] of von Petersdorff and Stephan, a multigrid method on graded meshes is considered for the hypersingular integral equation.