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ON PANTOGRAPH INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. The paper discusses the initial-value problem for the pantograph integro-differential equation, including as special cases the retarded functional-differential equation studied by Ockendon and Tayler [**17**], Kato and MacLeod [**11**] and the neutral differential equation studied by Kuang and Feldstein [**12**]. The main subjects of this paper are well-posedness of the initial-value problem, monotonicity and oscillation of the solution, unboundedness of the solution, and asymptotic stability of the solution, subject to different conditions.

1. Introduction. Let a be a complex constant and $\mu(q)$ and $\nu(q)$ complex-valued functions of bounded variation on $[0,1]$. The initial-value problem for pantograph integro-differential equations to be studied in this paper is of the form

$$\begin{aligned} y'(t) &= ay(t) + \int_0^1 y(qt)\,d\mu(q) \\ &\quad + \int_0^1 y'(qt)\,d\nu(q), \quad t > 0, \quad y(0) = y_0 \end{aligned} \tag{1.1}$$

where the integrals being considered are of Riemann-Stieltjes type, although most results of this paper still hold if $\mu(q)$ and $\nu(q)$ are replaced by complex-valued measures on $[0,1]$. The term *pantograph* comes from Ockendon and Tayler [**17**] and Iserles [**7**].

The pantograph integro-differential equation includes many interesting equations studied before. In the case $d\mu(q) = b\delta(q-p)\,dq$, $d\nu(q) \equiv 0$, where $p \in (0,1)$ and $\delta(\cdot)$ is a Dirac function, problem (1.1) can be written as

$$y'(t) = ay(t) + by(pt), \quad t > 0, \quad y(0) = y_0, \tag{1.2}$$

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