JOURNAL OF INTEGRAL EQUATIONS AND APPLICATIONS Volume 5, Number 2, Spring 1993

SOME EXISTENCE RESULTS FOR NONLINEAR INTEGRAL EQUATIONS VIA TOPOLOGICAL TRANSVERSALITY

R.B. GUENTHER AND J.W. LEE

ABSTRACT. Existence results are established for nonlinear integral equations of Hammerstein and Urysohn type. The results complement and extend related work in the field. A principal feature of the paper is its rather easy proofs which are based on topological transversality theory rather than degree theory.

1. Introduction. In this paper we establish some existence results for nonlinear integral equations of Hammerstein and Urysohn type. The results and proofs complement and extend similar results in the literature; see, for example, [6, 7]. A principal feature of the paper is the rather easy proofs that are based on a more elementary topological structure than is usually used. We base our discussion on the topological transversality theory of A. Granas [3] rather than on the Leray-Schauder degree. The more elementary point of view of topological transversality has led to many new results about nonlinear differential systems; see [2, 4, 5] for an overview. However, the methods of topological transversality have not been used much (perhaps at all) in the treatment of nonlinear integral equations. We hope this discussion will stimulate further work.

The development of a topological degree, such as the Leray-Schauder degree, requires substantial and rather sophisticated preliminaries. In contrast, the results typically needed from topological transversality theory require nothing more demanding than Urysohn's lemma in a metric space and a few standard compactness arguments. Full proofs of the results summarized below may be found in [4, p. 14–15] or in [1, p. 57–60], which also contains further theory. In applications, some maps must be known to have nonzero degree or to be essential, in the case of topological transversality. In either approach, the Schauder

Received by the editors on January 7, 1993. Partially supported by the Office of Naval Research Grant Number ONR: N0014-92-J-1226.

Copyright ©1993 Rocky Mountain Mathematics Consortium