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UNCONVENTIONAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. A simple method for the solution of second-kind singular integral equations with negative index is investigated. It makes use of Gaussian quadrature that is not of the type suggested by the theory. The major advantage is its simplicity. The error analysis shows that under reasonable assumptions on the smoothness of the solution, the proposed method is convergent. Numerical experiments reveal a higher convergence rate than the one obtained theoretically.

1. Introduction. In the recent literature, the numerical solution of singular integral equations (SIE's) has received considerable attention. It is possible to reduce SIE's to Fredholm integral equations, but in practice direct methods are preferred. The unknown function is replaced by the product of a smooth function times the fundamental function of the problem, with the latter taken as the weight of a quadrature rule. For variable coefficient SIE's, the weight function is nonclassical and the nodes and weights of the quadrature rule must be constructed from scratch. For constant coefficient SIE's, this reduces to Jacobi quadrature.

In this paper we want to analyze the replacement of the possibly nonclassical weights and nodes by the weights and zeros of the Chebyshev polynomials. This is a simpler approach than the standard one. It also has the basic advantage that in doubling the size of the system, the values of the kernel evaluated at the earlier run can be reused. It may also lead to a fast method for second kind SIE's. Recently a fast method has been proposed for first kind equations [**11**], but for second kind equations, one of the difficulties seems to be related to the asymmetry of the Jacobi nodes. In [**21**] an algorithm which uses arbitrary nodes is proposed, but no error analysis is provided. Here we consider

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This paper is dedicated to the memory of my parents, Lucia Corazza and Ernesto Venturino.

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