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STURM-LIOUVILLE PROBLEMS AND HAMMERSTEIN OPERATORS

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ABSTRACT. It is shown that a generally complex-valued function of a real variable is a solution of a classical Sturm-Liouville eigenvalue problem if and only if a related two-parameter eigenvalue problem for a pair of integral operators, one of which is of Hammerstein type, admits a real solution belonging to a cone in a Krein space.

1. Introduction. Let $q, w : [a, b] \equiv I \to R$; $q, w \in L[a, b]$ where a, b are finite real numbers. We define the sets E^0, E^+, E^-, respectively, by $\{x \in I : w(x) = 0\}$, $\{x \in I : w(x) > 0\}$, $\{x \in I : w(x) < 0\}$ and we assume that $\mu(E^0) = 0$, $\mu(E^+) > 0$, $\mu(E^-) > 0$, where μ is Lebesgue measure.

We now consider the Dirichlet problem associated with the Sturm-Liouville equation

$$-y'' + q(x)y = \lambda w(x)y, \tag{1.1}$$

on $a < x < b$, where

$$y(a) = y(b) = 0. \tag{1.2}$$

The existence and asymptotic behavior of the real eigenvalues of this problem has been treated elsewhere and we refer the interested reader to [**1, 3**] for details. We emphasize here that there are no sign restrictions on the coefficients q, w above. Of specific interest here is the existence or nonexistence of *nonreal* eigenvalues and their related eigenfunctions. This question dates back to the pioneering studies of Otto Haupt and Roland Richardson, see the survey paper [**5**] for these and other

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