JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 4, Number 1, Winter 1992

ON NONCOMPACT HAMMERSTEIN INTEGRAL EQUATIONS AND A NONLINEAR BOUNDARY VALUE PROBLEM FOR THE HEAT EQUATION

P.P.B. EGGERMONT

ABSTRACT. We discuss the solvability of noncompact Hammerstein integral equations, related to Volterra as well as Wiener-Hopf equations. Usually the solvability is well understood only in L^2 setting, e.g., if the integral operator is positive definite and the nonlinearity is monotone. We are interested in obtaining the L^∞ theory from the L^2 theory. This can be done by means of a result related to Hadamard's theorem, which allows us to consider the solvability of the linearizations of the Hammerstein equation; by means of a theorem in [**5**] concerning the spectra of convolution-like operators on Lebesgue spaces; and by means of a compactness argument involving the strict topology on L^∞. We apply this theory to the study of the solvability of the heat equation on a half space with (mildly) nonlinear heat radiation on the boundary.

1. Introduction. In this paper we study the solvability of noncompact Hammerstein integral equations, prototypical of which are nonlinear convolution equations on Lebesgue spaces. Usually, equations like these are well understood in L^2 setting, and it is desirable to obtain an L^∞ theory without additional conditions. Typically, a satisfactory L^∞ theory is helpful when we want to establish uniform error estimates of numerical methods for these equations, most notoriously for solutions obtained by Galerkin methods, but sometimes the intrinsic interest is in the L^∞ theory to begin with, such as problems related to the heat equation. We consider only *mild* nonlinearities, i.e., nonlinearities with a reasonable *Lipschitz constant.* This allows us to linearize the Hammerstein equation and leads to problems about the spectra of integral operators on Lebesgue spaces. An indispensable technical device turns out to be the *strict* topology on L^∞, see [**8**] and, particularly, [**2, 3, 4**]. For an application of some of these matters in a related context, see [**1**].

1980 *Mathematics subject classification* (1985 Revision). 45G10, 35K60, 45E10, 47H99.